Different Game Concepts in Internet Economics

Rationality Concepts

- Individual
  - Nash Equilibrium
  - Envy-free: cake cutting (locally envy free)
- Market model
  - Incentive compatibility
    - Keyword Auction: GSP
    - Forward Looking Nash Equilibrium and insight
  - Non-arbitrage
  - General equilibrium

Individual Rationality

- Self motivation to maximize one's own
  – Utilities, or
  – Happiness, or
  – Profit

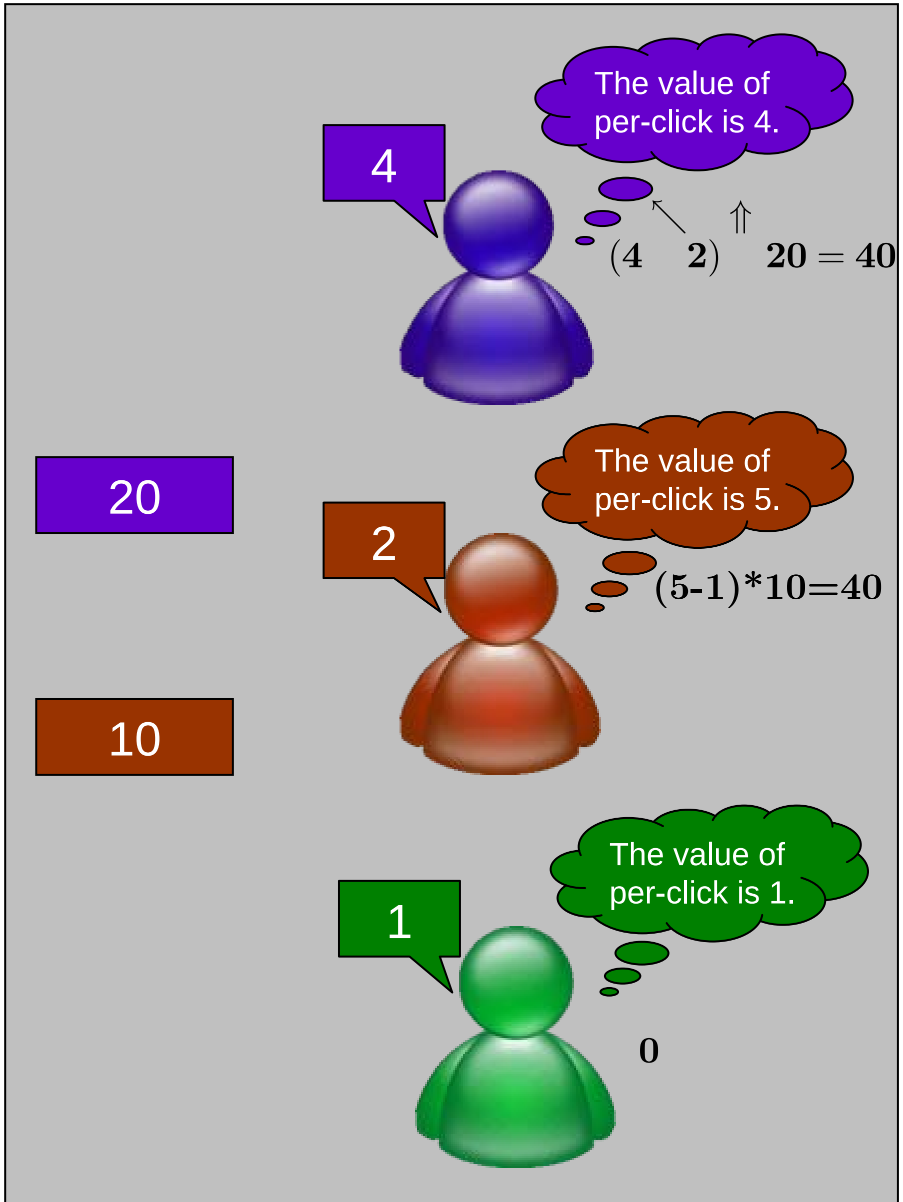
Nash Equilibrium

- A stable solution where
  - everyone chooses its own best response

Generalize Second Price Auction

- Each slot generates a fixed number of clicks
- Each advertiser has a value for one click
- Highest bidder gets the best slot, paying the second highest bidding price
- i-th highest bidder gets the i-th best one, paying the i+1st highest bidding price

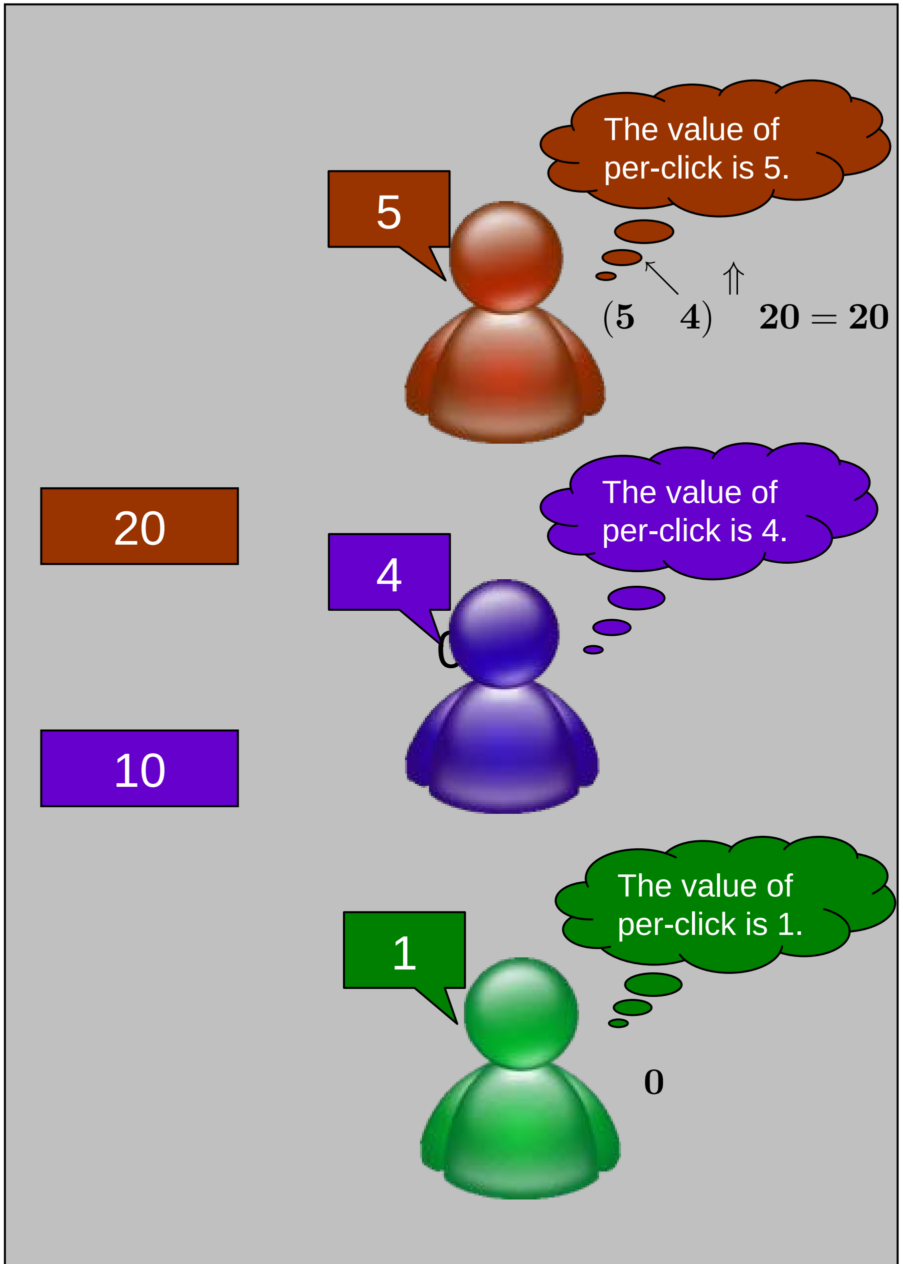
A Pure Nash Equilibrium



- BLUE gets 20 clicks
  - Pays 2 each
  - Profit = (4-2)*20 = 40
- RED gets 10 clicks
  - Pays 1 each
  - Profit = (5-1)*10 = 40
- GREEN gets 0 clicks
  - Pay nothing
  - Profit = 0

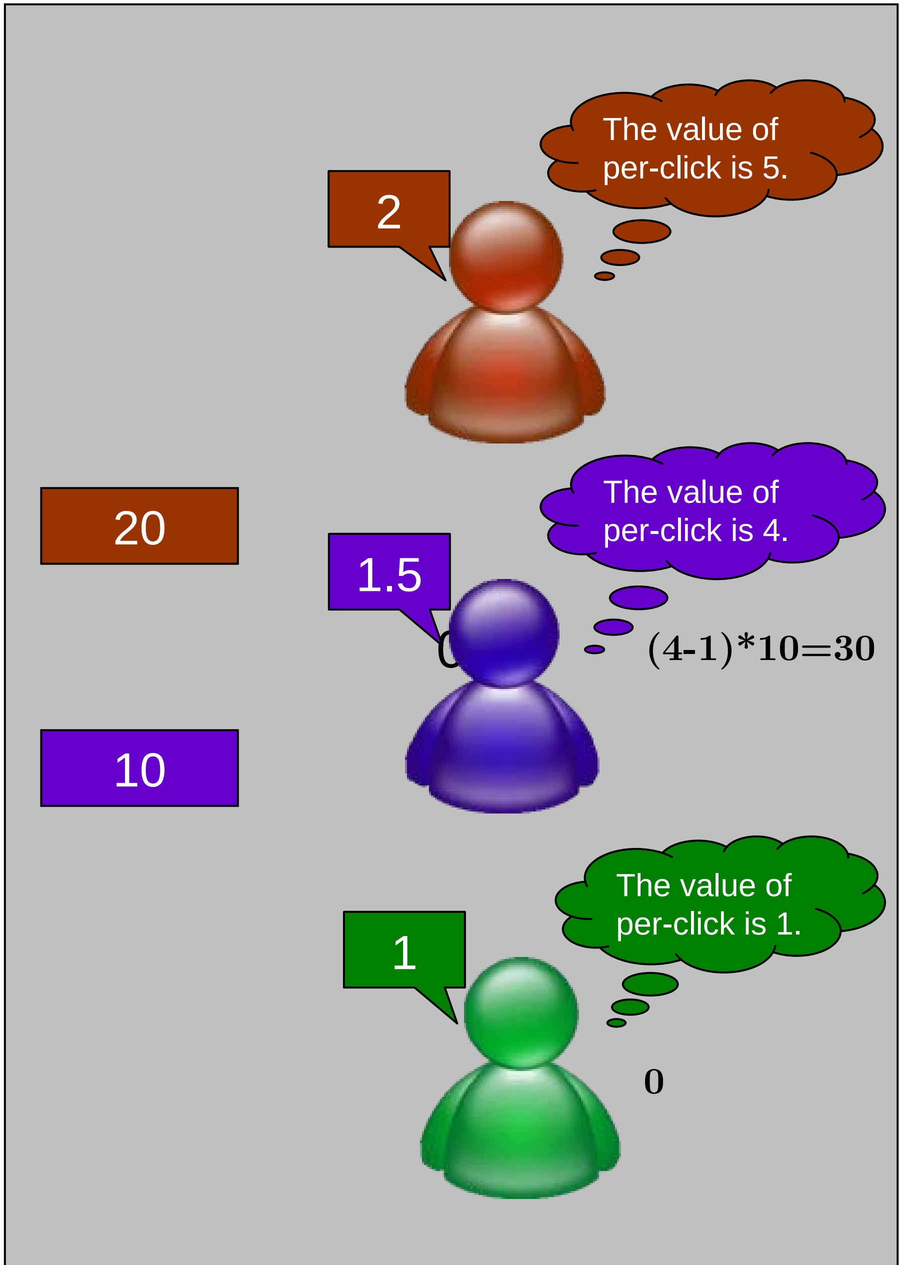
- No one gets better profits by changing its bid.

RED cannot improve



- If RED bids more than 4, while others do not change
  - it gets 20 clicks
    - Pays 4 each
    - Profit= (5-4)*20 = 20
  - which is less than 40, his original profit.
- If it bids less than 1, it gets nothing, and its profit is zero.

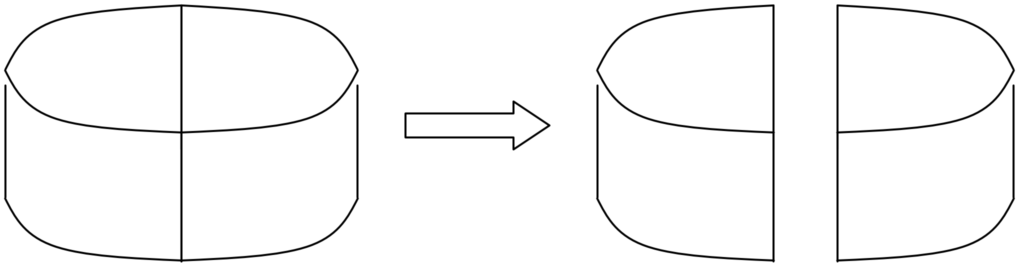
BLUE cannot improve



- If BLUE bids more than 2, while others do not change
  - it gets 10 clicks
    - Pays 1 each
    - Profit= (4-1)*10 = 30
  - which is less than 40, his original profit.
- If it bids less than 1, it gets nothing, and its profit is zero.

Concept of Envy-Free

- Cake-cutting:
  - Two children, Alex and Bob, to share a cake



Individual Preference

- Some wants Strawberries with his/her piece of the cake
- Some likes chocolate on the cake

- Could we cut it in a way so that everyone can get his/her best choice?

How to make a fair division ?

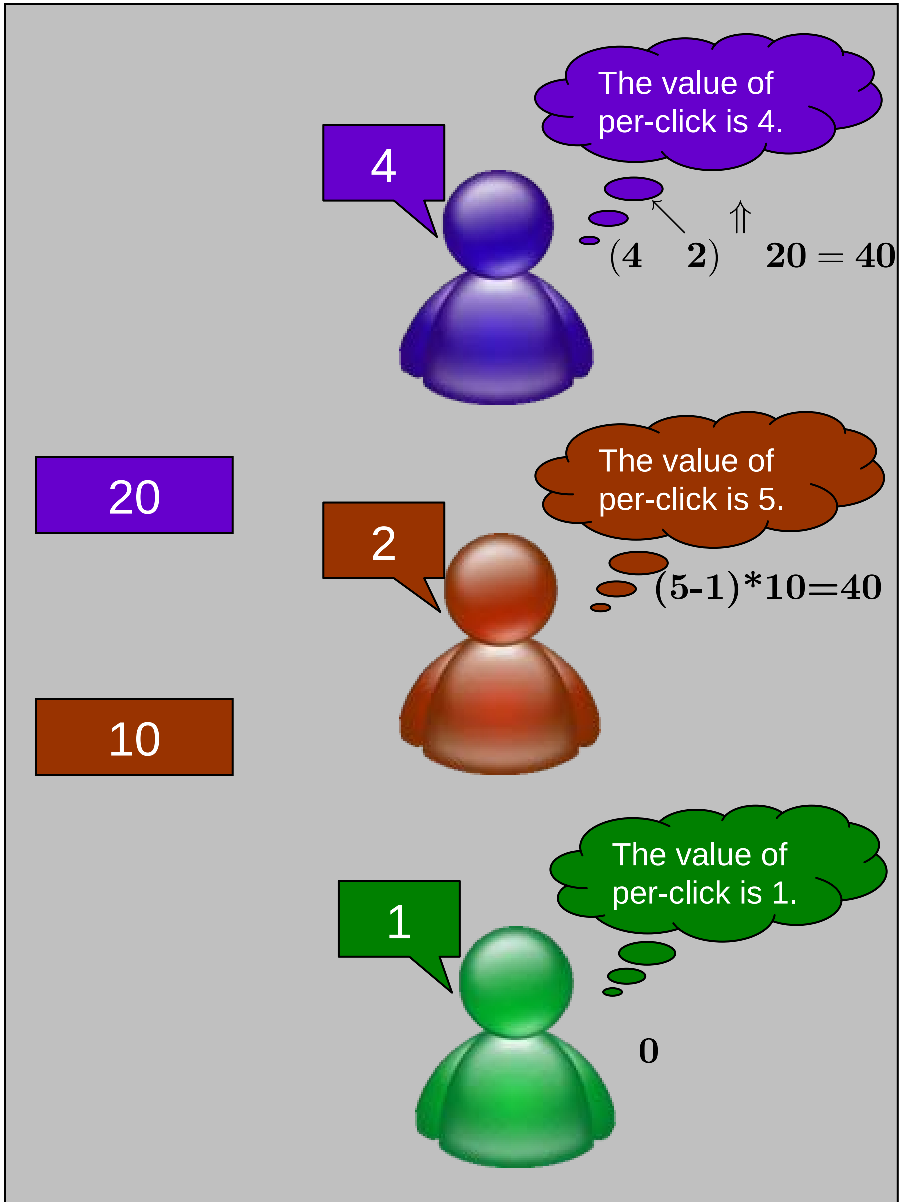
- Envy-free:
  - Cut the cake in two pieces, one for each
  - Neither Alex nor Bob prefers the other piece.
- Solution:
  - Cut and Choose:
    - Alex cuts and Bob chooses first

Envy-Free Solution in GSP

- Envy-free:
  - No bidder would like to exchange its bid with any other bidders

- Locally Envy-free:
  - No bidder would like to exchange its bid with the one immediately above it.
- Proposed for generalized second price auction by
      - » **Benjamin Edelman**, **Michael Ostrovsky**, and **Michael Schwarz**
  - Equivalent to Symmetric Nash equilibrium
      - » **Hal Varian**

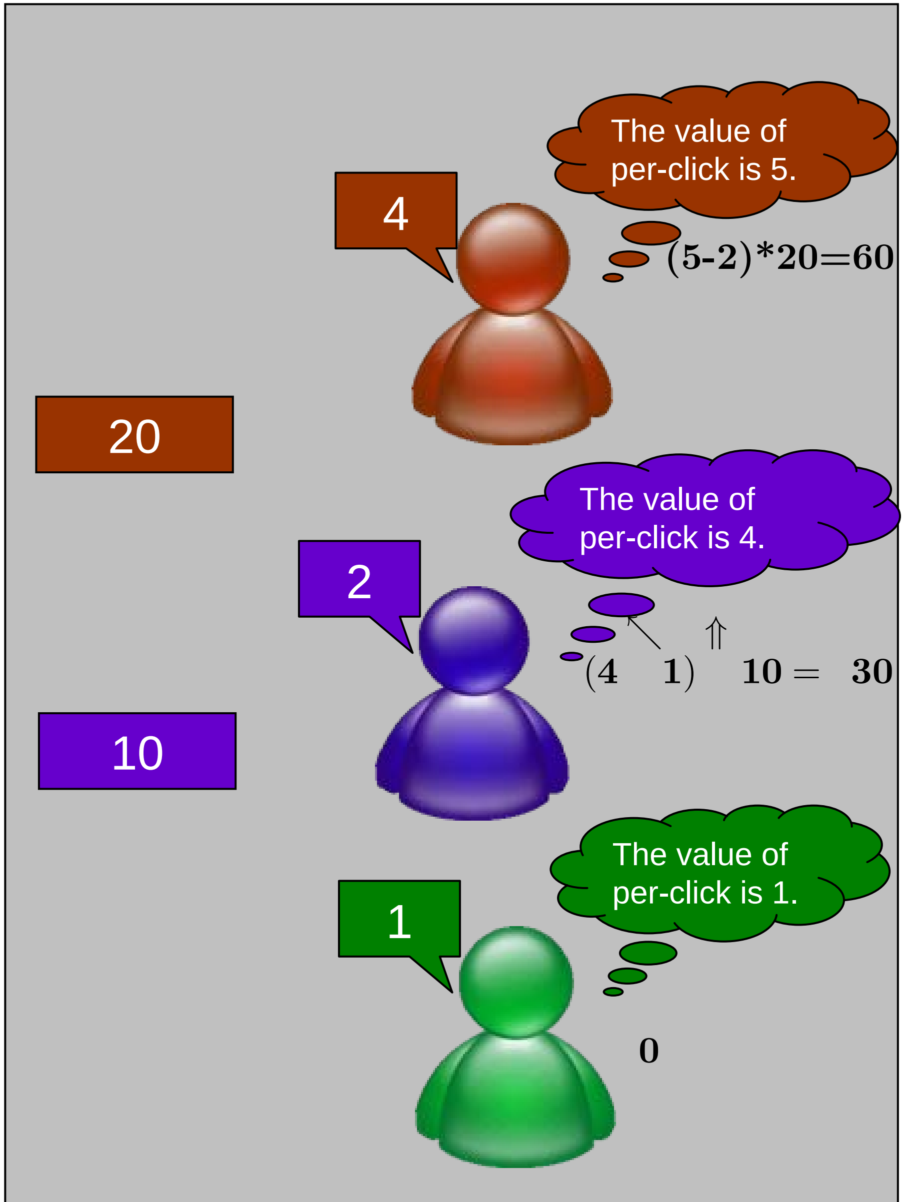
Consider A Pure Nash Equilibrium



- BLUE gets 20 clicks
  - Pays 2 each
  - Profit = (4-2)*20 = 40
- RED gets 10 clicks
  - Pays 1 each
  - Profit = (5-1)*10 = 40
- GREEN gets 0 clicks
  - Pay nothing
  - Profit = 0

- No one gets better profits by changing its bid.

BLUE and RED exchange bids

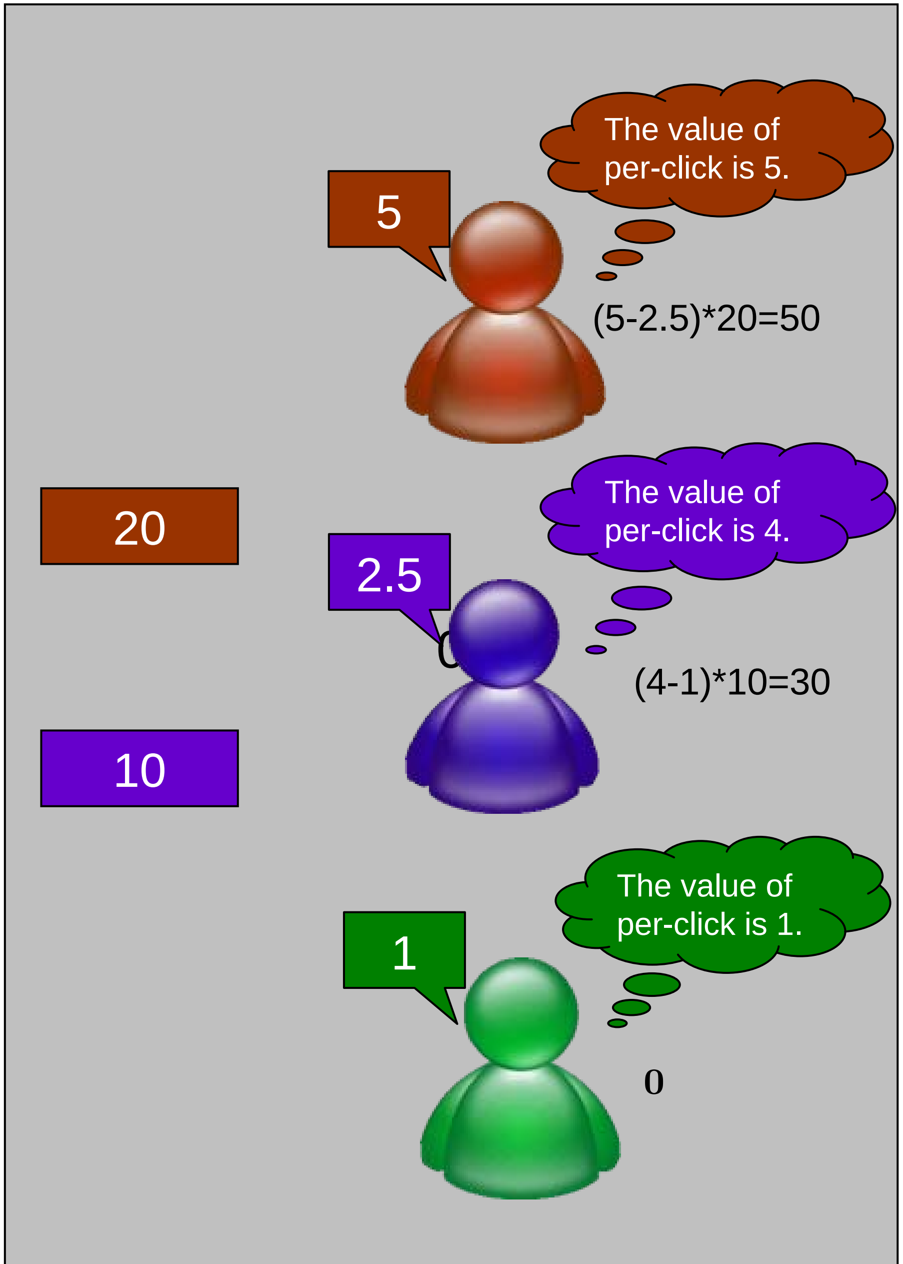


- BLUE gets 10 clicks
  - Pays 1 each
  - Profit = (4-1)*10 = 30
- RED gets 20 clicks
  - Pays 2 each
  - Profit = (5-2)*30 = 60
- GREEN gets 0 clicks
  - Pay nothing
  - Profit = 0

- RED improves its profit
  - RED envies BLUE

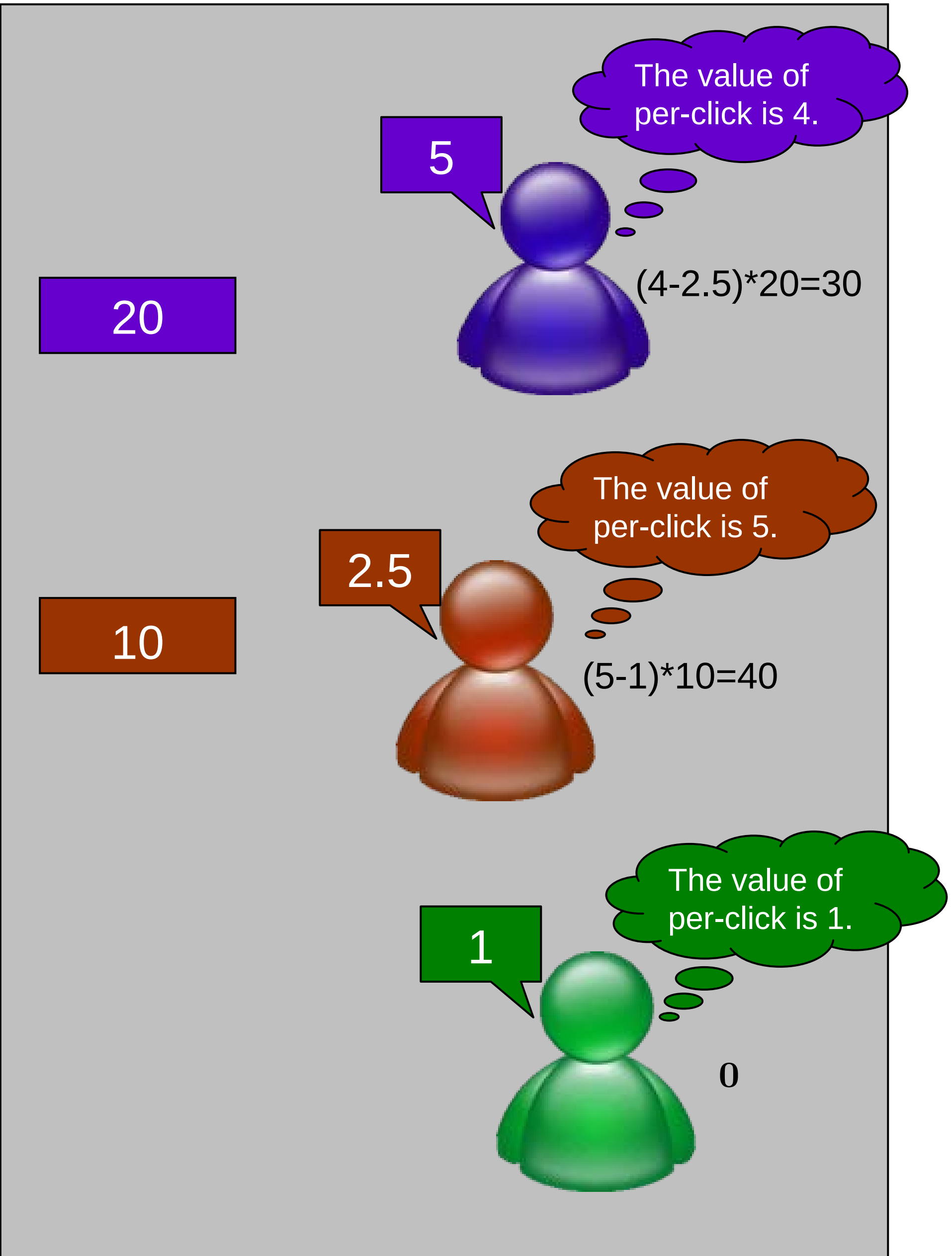
- Therefore, not envy-free

An Envy Free Solution



- RED gets 20 clicks
  - Pays 2 each
  - Profit= 50
- BLUE gets 10 clicks
  - Pays 1 each
  - Profit= 30

An Envy Free Solution



- If RED and BLUE exchange bids
- BLUE gets 20 clicks
  - Pays 2.5 each
  - Profit= 30
  - which is no more than 30, his original profit.
- RED gets 10 clicks
  - Pays 1 each
  - Profit= 40
  - which is less than 50, his original profit.

Group Rationality

- Cooperative, Negotiated outcome that are mutually beneficial for all agents.

Exercise

- Consider a project group of two people.
- The project tasks are:
  - Coming up a project plan
  - Doing the analysis
  - Programming
  - Writing up the final report
- How do you two decide on the parts you work on and how to allocate percentages of the grades fairly?

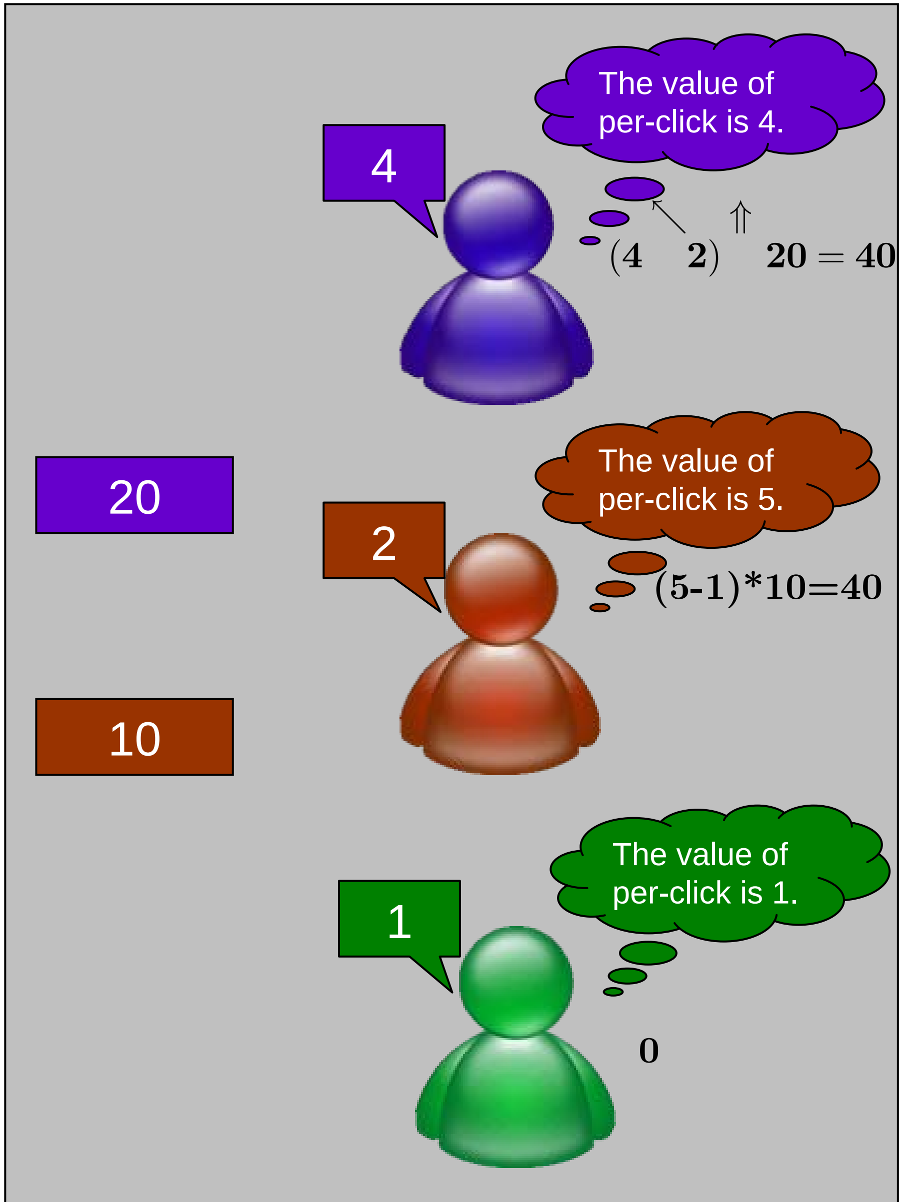
Group Rationality

- Von Neumann studied the equilibrium concept for two player zero sum game, showing its existence,
  - which is known now equivalent to the linear programming duality theorem
  - Therefore, a FAST (polynomial time) algorithm exists.
- He moved on, together with Morgenstern, to the study of cooperative games for games of multiple participants.

Core in Cooperative Games

- Two major types of cooperative games
  - Non-Transferable utility
    - Members of a subgroup can coordinate their decisions but each retains whatever he/she gets
  - Transferable utility
    - Members of a subgroup can transfer their revenues among themselves
- Core
  - A distribution of incomes to all members of the game such that
    - No subgroup can break away from the grand coalition such that everyone in the subgroup gains.

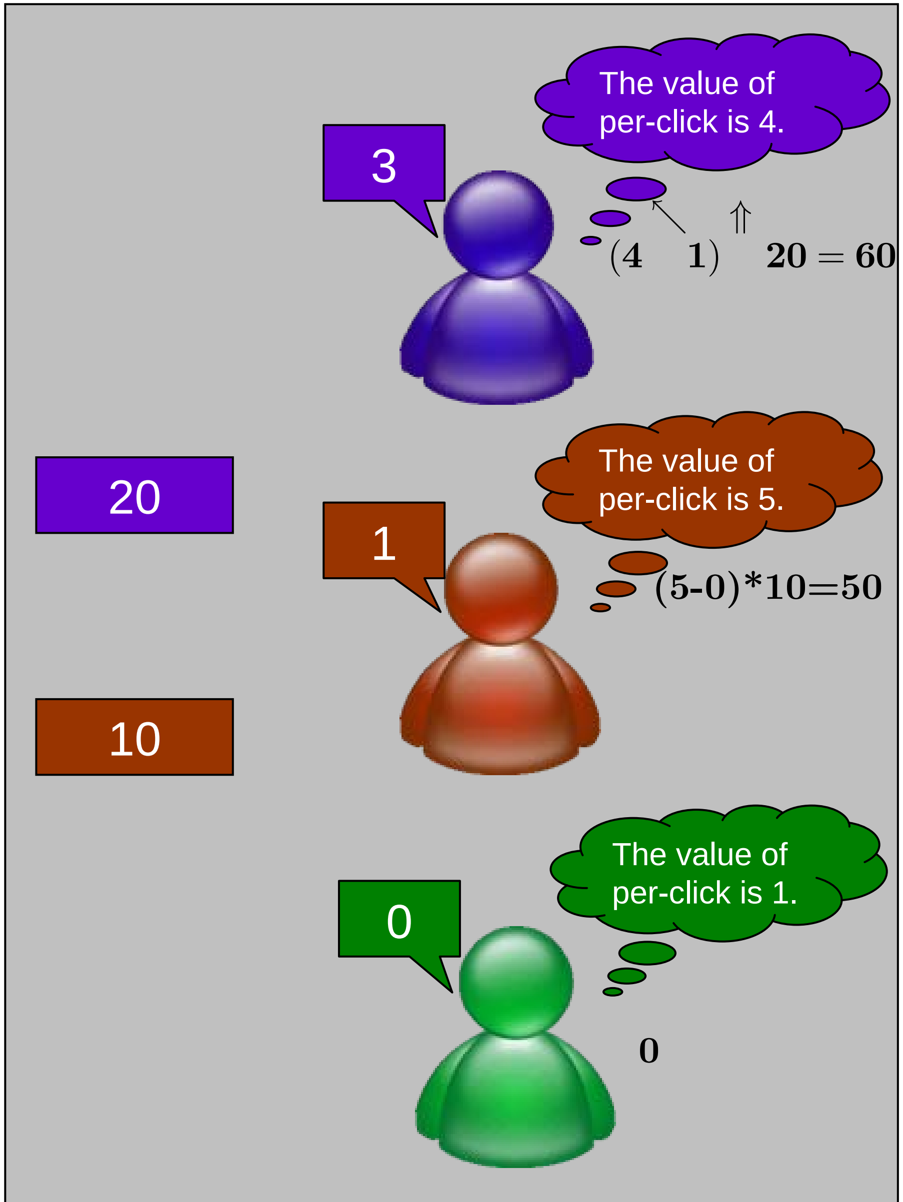
Example of GSP: A Nash equilibrium not in the Core (transferable utilities)



- Nash Equilibrium:
- BLUE gets 20 clicks
  - Pays 2 each
  - Profit = (4-2)*20 = 40
- RED gets 10 clicks
  - Pays 1 each
  - Profit = (5-1)*10 = 40
- GREEN gets 0 clicks
  - Pay nothing
  - Profit = 0

- No one gets better profits by changing its bid.

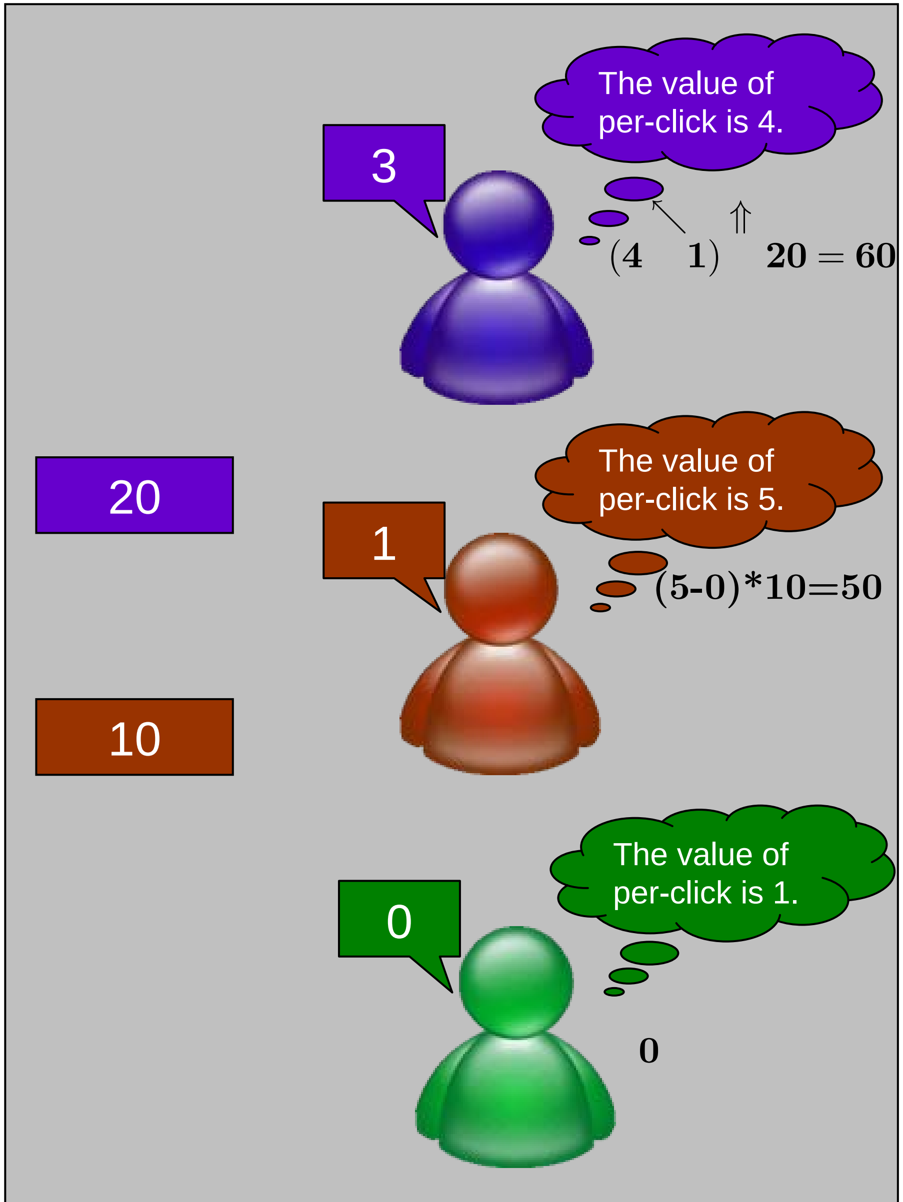
Coordinated Bids



- Coordinated biddings:
- BLUE gets 20 clicks
  - Pays 1 each
  - Profit = (4-1)*20 = 60
- RED gets 10 clicks
  - Pays 0 each
  - Profit = (5-0)*10 = 50
- GREEN gets 0 clicks
  - Pay nothing
  - Profit = 0

- One way to improve profit of all is for RED and BLUE to give 5 each to GREEN.

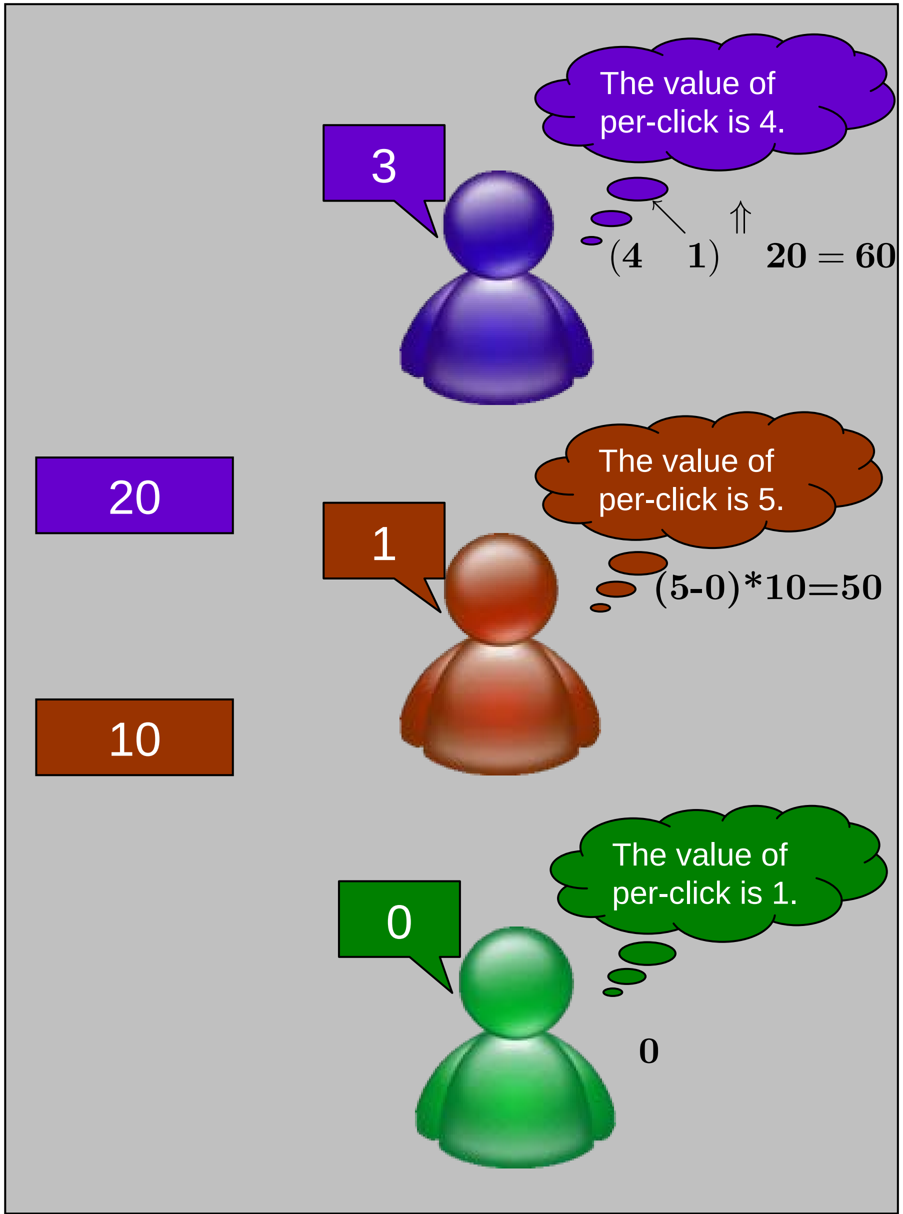
…and Transfer



- GREEN benefit from transfer from RED and BLUE (5 each)

- BLUE gets 20 clicks
  - Pays 1 each
  - Profit = (4-1)*20-5 = 55
- RED gets 10 clicks
  - Pays 0 each
  - Profit = (5-0)*10 -5 = 45
- GREEN gets 0 clicks
  - Pay nothing
  - Profit = 5+5 = 10

non-transferable utility



- Coordinated biddings:
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  - Pays 0 each
  - Profit = (5-0)*10 = 50
- GREEN gets 0 clicks
  - Pay nothing
  - Profit = 0

- All benefit except GREEN who does not lose.

Exercise

- What is a solution in the Core of Generalized Second Price Auction Protocol?
  - Is the core always non-empty
  - How to decide if the core is not empty.
- Give examples or proofs for your claims.
  - For transferable utility
  - For non-transferable utility

Market Rationality

- Monetary measure is used for measure of utilities and winners/losers are determined by market principles.
  - Incentive compatibility
  - Non-arbitrage
  - General equilibrium

VCG Mechanism

- Generalize Vickrey Auction for multiple different item auction
- Protocol:
  - Each player in the auction pays the opportunity cost that their presence introduces to all the other players.
- Example:
  - Auction a pen and a pencil
  - Three bidders A, B, C
  - A bids $5 for a pen; B bids $2 for a pencil; C bids $6 for both
  - Outcome: A wins a pen and pays $4; B wins a pencil and pays $1

VCG mechanism is truthful: It is optimal for a player to bid its true value of the item.

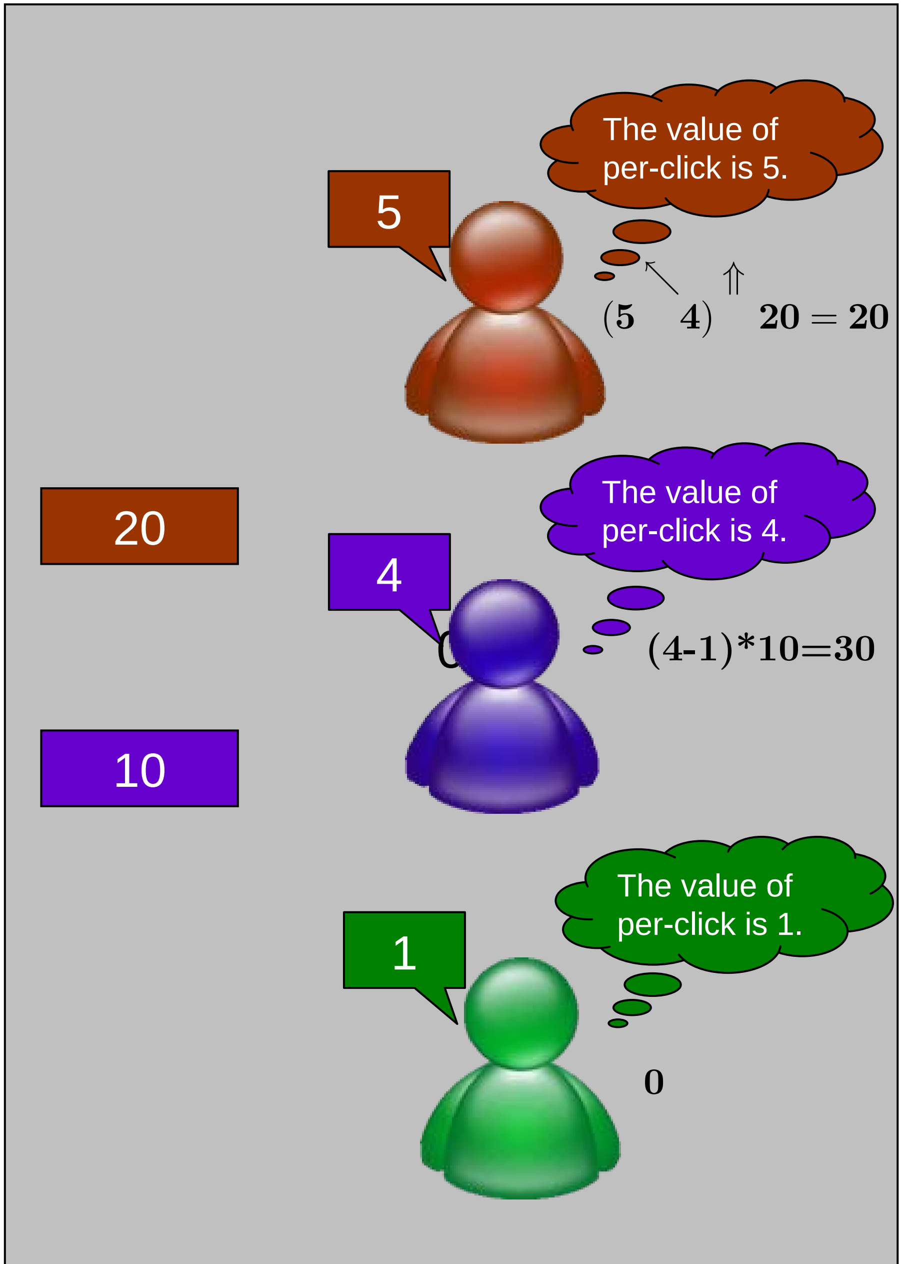
Why Truthful is Important?

- Nash equilibrium
  - A set of strategies that make none willing to change.
- Revenue Equivalence Theorem (Myerson)
  - Under a wide range of conditions
  - a protocol N that guarantees a Nash equilibrium can be transferred to a protocol T that is truthful such that
    - N and T have the same payment for everyone

Unfortunately

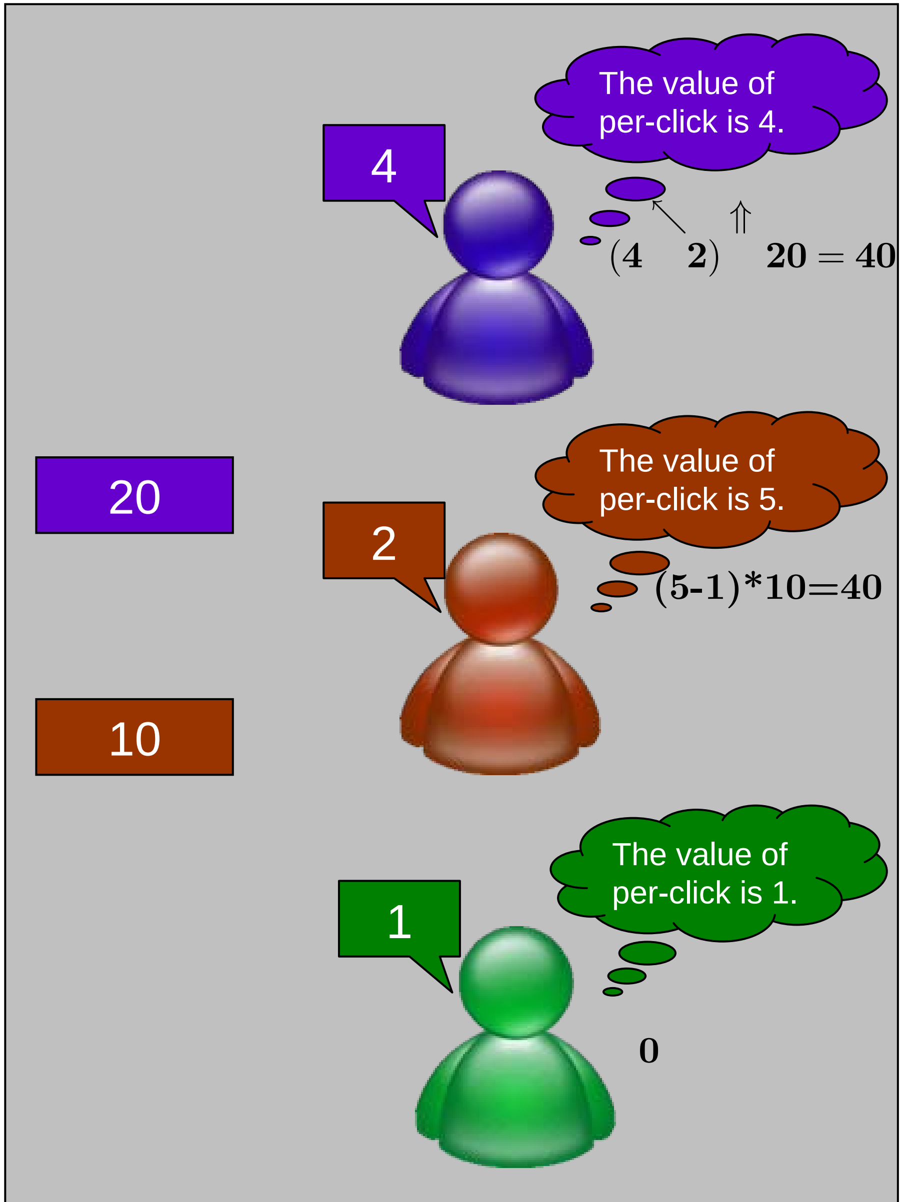
- VCG was not used in Sponsored Search Market
- In fact, no truthful protocol for its sponsored search auction.
  - (of course, VCG is too complicated for an average participant to understand?).
- Generalize Second Price Auction
  - was used instead

GSP is not truthful



- Two slots with 20/10 clicks
- Three bidders with true values 5/4/1
- Value5 gets 20 clicks
  - Pay 4 each
  - Utility= (5-4)*20 = 20
- Value4 gets 10 clicks
  - Pay 1 each
  - Utility= (4-1)*10 = 30

RED would bid lower



- Value5 gets 10 clicks
  - Pay 1 each
  - Utility = (5-1)*10 = 40
- Value4 gets 20 clicks
  - Pay 2 each
  - Utility = (4-2)*20 = 40
- Value5 gets better by changing its bid

What is a good Nash equilibrium?

- Studied by Various People, most noticeably:
- Varian (2006):
  – Coin the phrase position auction, and introduced the concept of Symmetric Nash Equilibrium (SNE), proved that VCG generates a total revenue that is a lower bound for SNE.
- Edelman Ostrovsky and Schwarz (2006)
  – introduced the concept of locally envy free equilibrium (LEFE).
- It has been known that LEFE and SNE are equivalent.

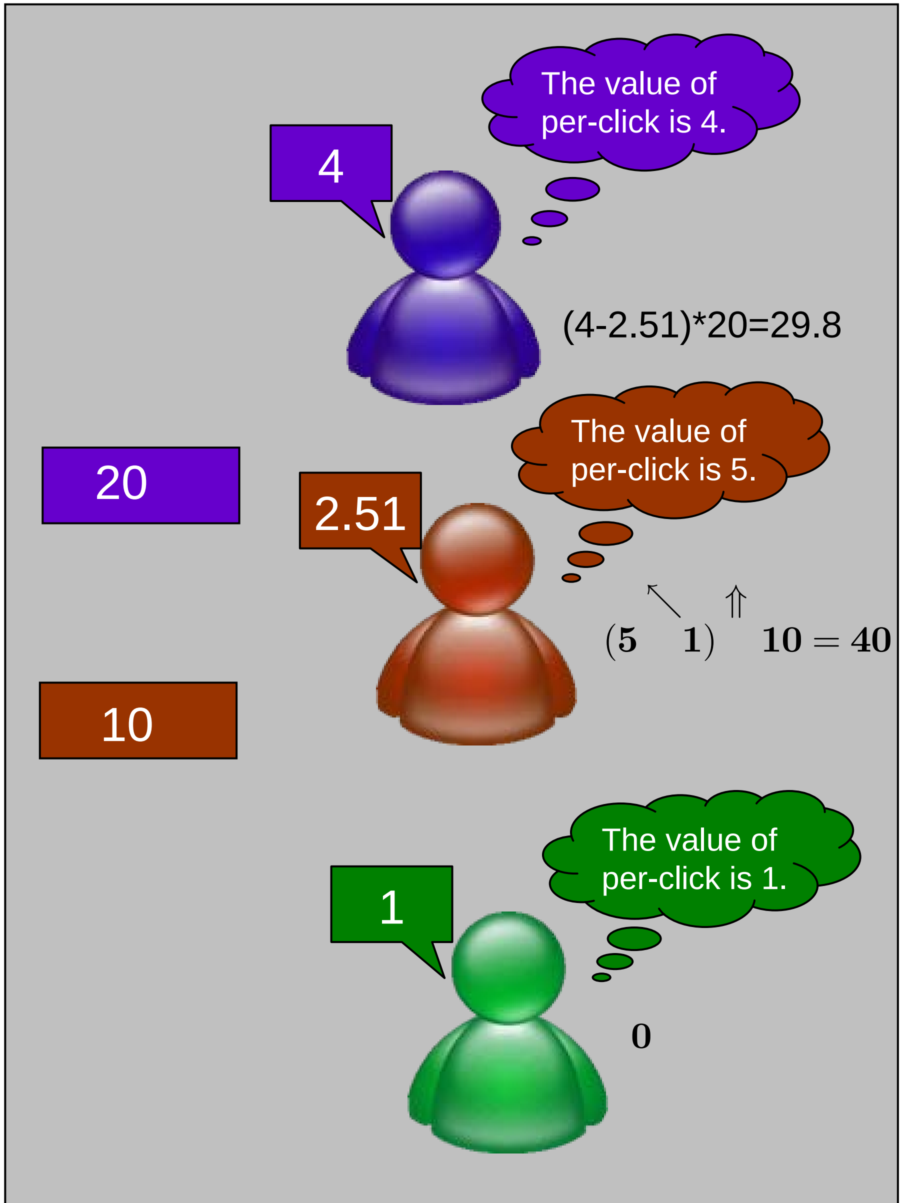
- Aggarwal, Goel and Motowani., considered laddered auction, equivalent to VCG payment protocol.
  - Relying on the seller's goodwill to implement it

Forward Looking Nash Equilibrium

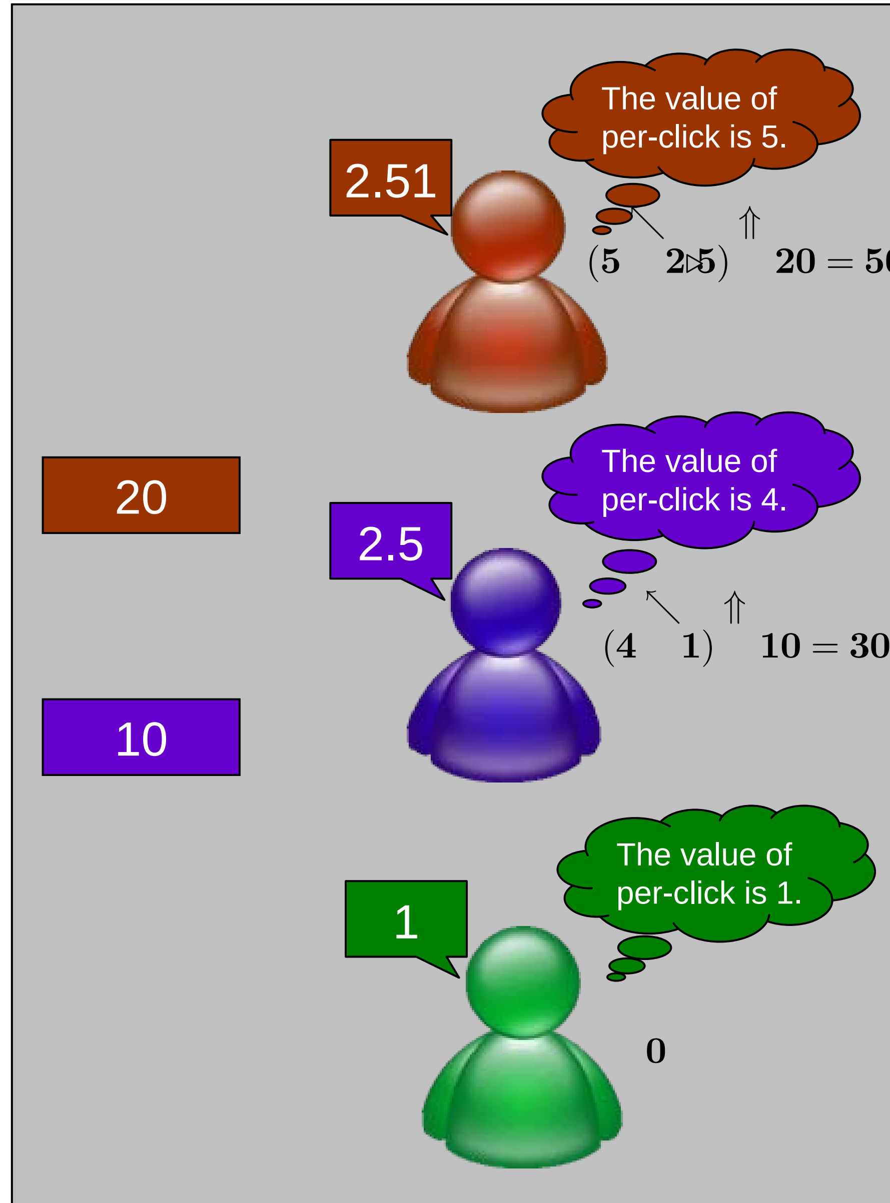
– Bu, D, Qi

– An equivalent greedy strategy proposed by Cary, et al.

The 2nd slot winner try to manipulate

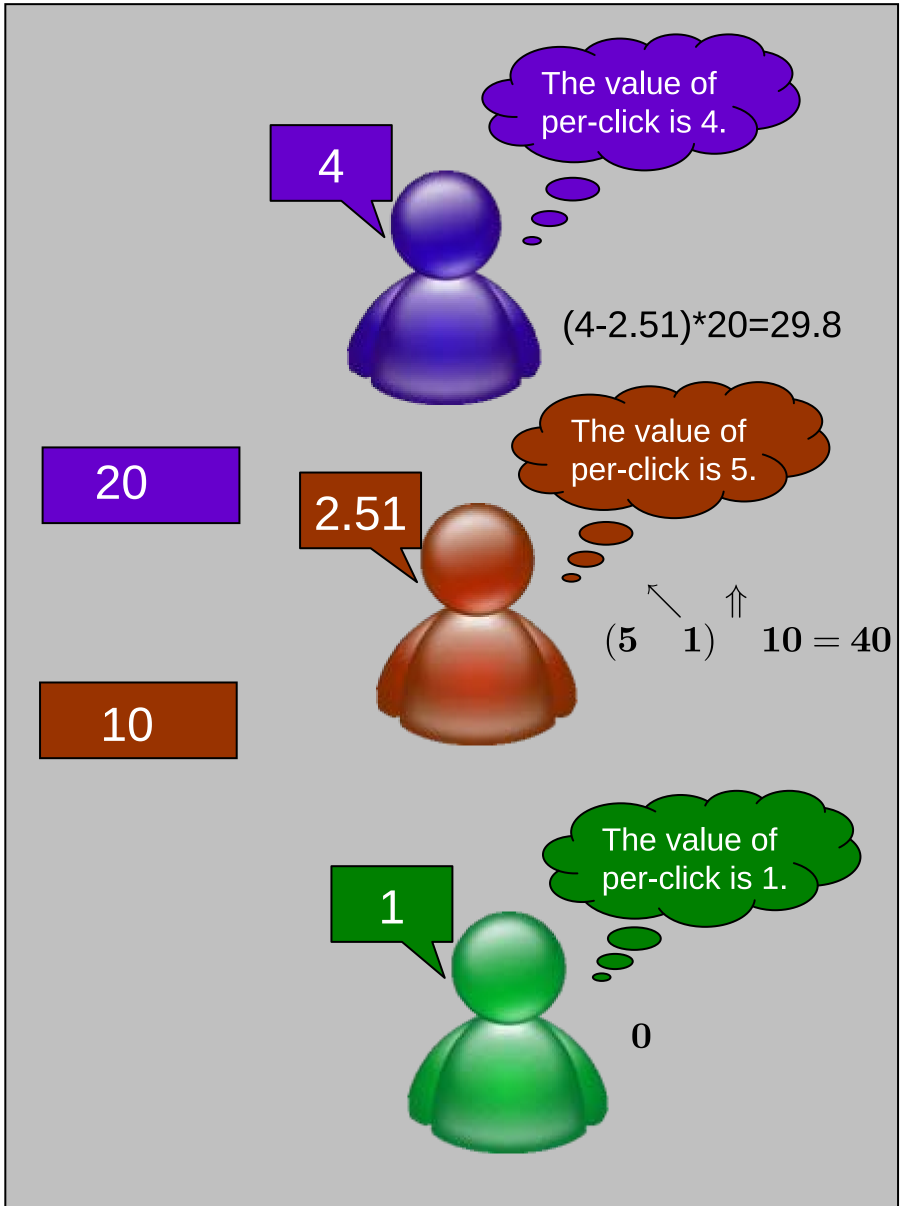


The first slot winner moves down to optimize its utility



Forward-looking Condition

- Given the bids of other players. Let the best chosen slot of bidder i be k.
  - In the example, RED at the second slot if it bids between 1+ to 4-.



What to choose between 1+ and 4-?

- Let it bids b:
  - current utility is $c_k(v^i-b^{k+1})=10*(5-1)=40$
- – As it bids ***b***, other bidders may also change their bid so that the new allocated slot may be a smaller index $t < k$.

What to choose between 1+ and 4-?

- As it bids ***b***, other bidders may also change their bid so that the new allocated slot may be a smaller index $t < k$.
- The worst case is to bid just smaller than b: $b-\varepsilon$
  - The utility of bidder i will be at least $c_t(v^i-(b-\varepsilon))$.

In the example

$20*(5- (b-\varepsilon))$

What to choose between 1+ and 4-?

– The worst case is to bid just smaller than b: b-ε

– The utility of bidder i will be at least $c_t(v^i-(b-\varepsilon))$.

– We want choose b such that it is larger than the current utility: $c_k(v^i-b^{k+1})$

– Thus, the condition $c_t(v^i-b+\varepsilon) > c_k(v^i-b_{k+1})$.

Taking ε goes to zero, we have

– $c_t(v^i-b) \geqslant c_k(v^i-b_{k+1})$.

20*(5- b)) $\geqslant$ 10*(5-1)

RED will bid 3

Forward-looking Attribute

Given $\forall \mathbb{b}^{-i}$, $b^i \in \mathcal{M}^i(\mathbb{b}^{-i})$. Let $k = \mathcal{O}^i(b^i, \mathbb{b}^{-i})$ be the slot assigned to bidder i and its utility be u^i_k. Let the bid of the bidder assigned to slot $k+1$ be b_{k+1}.

Now let the assigned slot to bidder i be t after other bidders change their bids within the range of their optimal response: $\mathcal{O}^i(\mathcal{M}^{-i}(\mathbb{b}^{-i}, b^i), b^i) = t$. Denote its utility as $\tilde{u}^i_t$. Then we have:

Lemma 3.3.4 (Forward looking attribute). *For all $t : t < k$, $u^i_k \leq \tilde{u}^i_t$ if and only if*

$$b^i \leq v^i - \frac{c_k}{c_{k-1}}(v^i - b_{k+1})$$

Forward-looking Nash Equilibrium

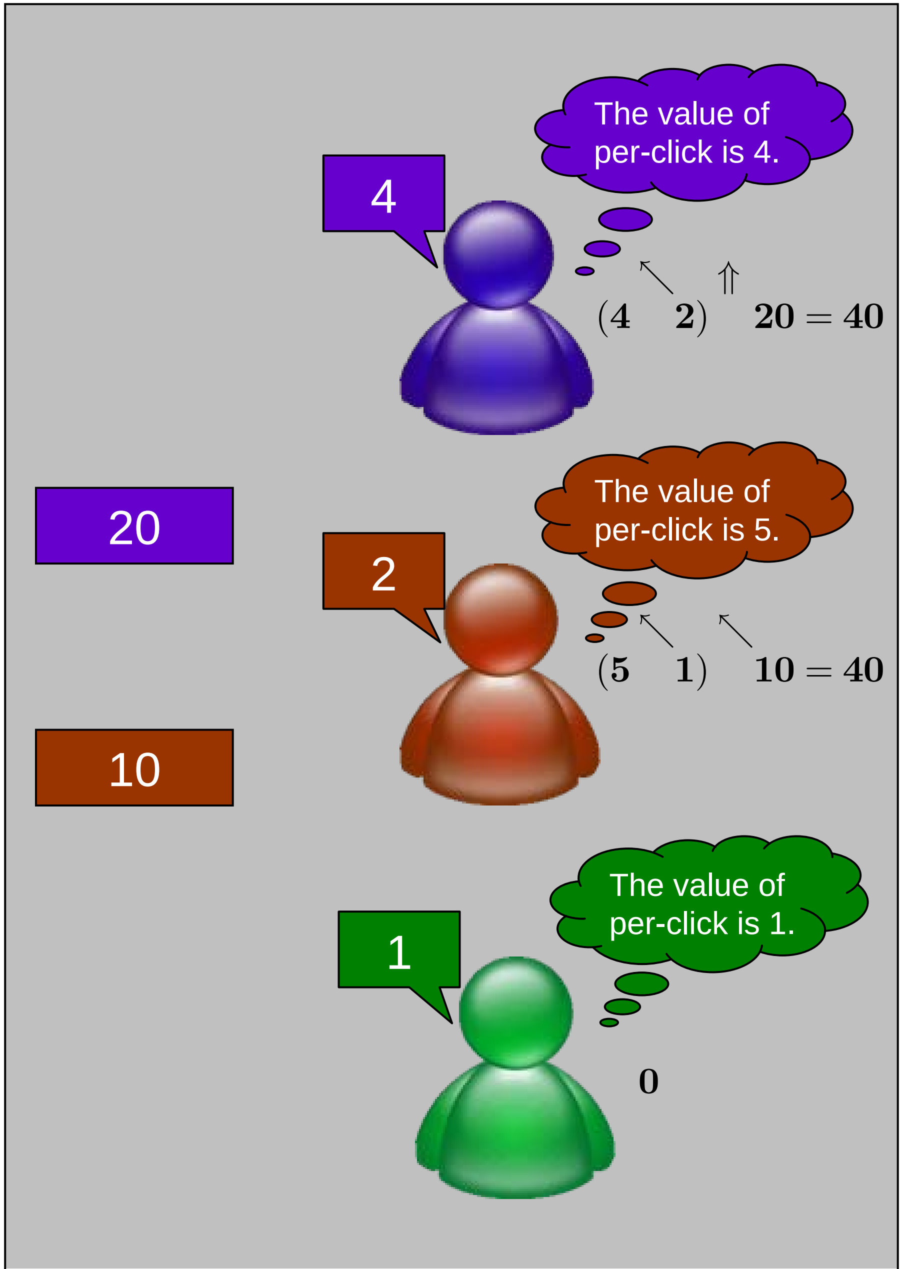
- Forward-looking response function

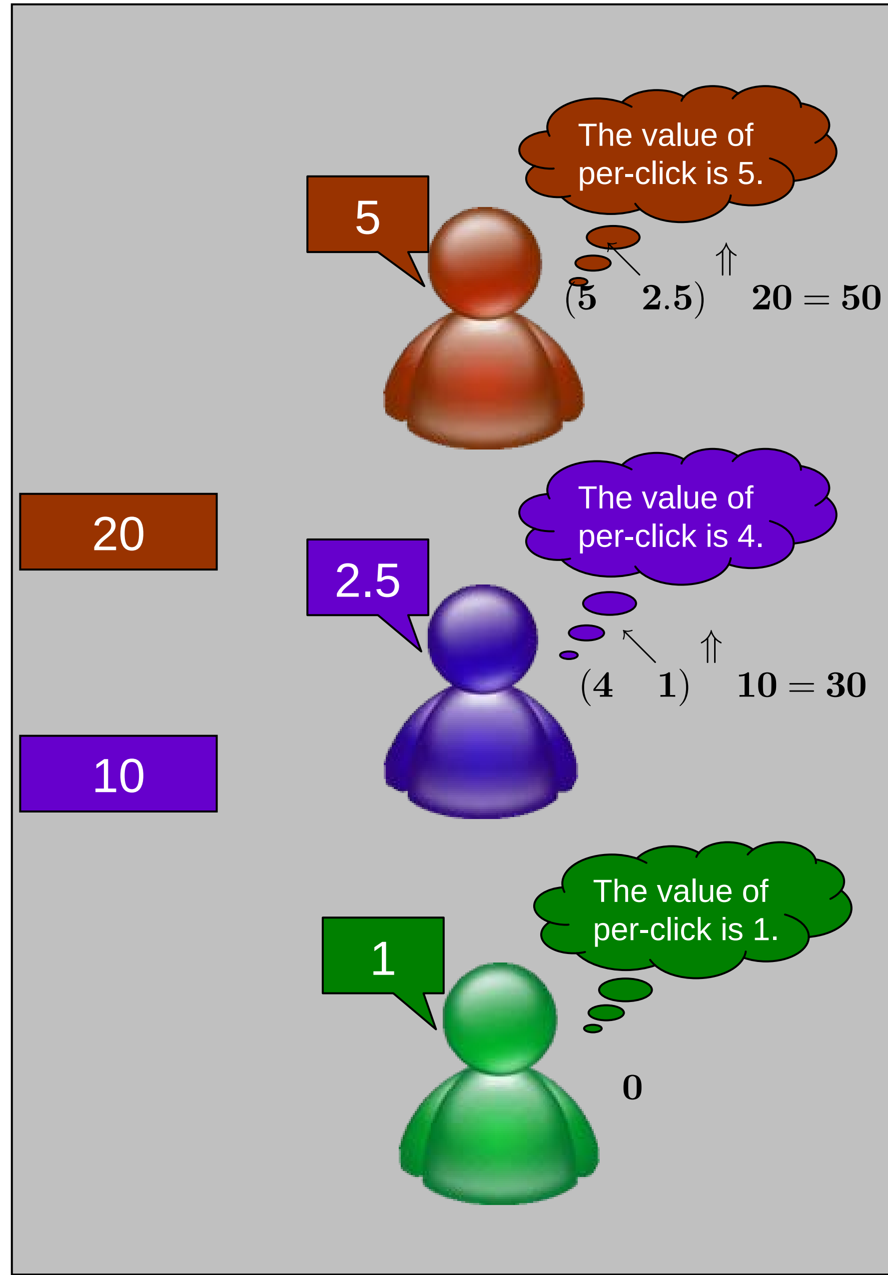
$$\mathcal{F}^i(\mathbb{b}^{-i}) = \begin{cases} v^i - \frac{c_k}{c_{k-1}}(v^i - b_{k+1}) & 2 \le k \le K \\ v^i & k = 1 \text{ or } k > K \end{cases}$$

- Forward-looking Nash Equilibrium

$$\begin{cases} b^i = v^i & \textit{for } i = 1 \textit{ and } i > K, \\ b^i = \frac{1}{\theta_{i-1}}\left[\sum_{j=i}^{K}(\theta_{j-1} - \theta_j)v^j + \theta_K v^{K+1}\right] & \textit{for } 2 \le i \le K. \end{cases}$$

Myopic Nash Equilibrium —— Forwarding-looking Equilibrium





Revenue Equivalence Theorem

- Any bidder's payment under the forward-looking equilibrium is equal to her payment under VCG mechanism for the auction.
- For sponsored search auction, the auctioneer's revenue in forward-looking equilibrium is equal to her revenue under VCG mechanism for the auction.
  - It is an RET though the conditions for it to hold were not known previously.

No-arbitrage Rationality

- There exists no risk-less profit opportunities in any stable market

Arbitrage in Sponsored Search Market

- Unfortunately this is not the case in sponsored search markets
  - Arbitrage does exist through not allowed to some extent
    - “AdSense”, “Google eat Google”,…
  - Bu, Deng and Qi, “Arbitrage Opportunities across Search Markets”, Workshop on Targeting and Ranking for Online Advertising.
- Interpretation:
  - Opportunities for further improvements of sponsored search market efficiencies

Multiple Markets(多元市场)

- M Search engine A_1, A_2, …, A_M
  - Each holds a GSP auction G_1, G_2, …, G_M.
  - Each auction has K slots
- N advertisers
  - Participate in all the auctions
  - True value: $V^{i,Gj}$
  - Bidding price: $b^{i,Gj}$

Forward looking response function in cross markets

$\forall i \in \mathcal{N} \cup \mathcal{A}_j$, given the other bidders' bidding set $\mathbf{b}^{-i,G_j}$, if bidder i prefers slot k, then bidder i's forward looking response function $\mathcal{F}^{i,G_j}(\mathbf{b}^{-i,G_j})$ is defined as

$$\mathcal{F}^{i,G_j}(\mathbf{b}^{-i,G_j}) = \begin{cases} v^{i,G_j} - \frac{\theta_k^{G_j}}{\theta_{k-1}^{G_j}}(v^{i,G_j} - b_{k+1}^{G_j}) & 2 \leq k \leq K \\ v^{i,G_j} & k = 1 \text{ or } k > K \end{cases} \tag{3.1}$$

- We identify two types of arbitrage behavior across sponsored search markets.
- Forward looking Nash equilibrium enables us to prove that they would improve auctioneers' revenues

Google

rose mask | Search | Advanced Search | Preferences

Web Results **1** - **10** of about **1,690,000** for **rose mask**. (**0.17** seconds)

Fresh **Rose Mask** - Face & Skin Care - Compare Prices, Reviews and **...**
Fresh **Rose Mask** - 5 results like the Fresh **Rose** Face **Mask** 100ml / 3.5oz, Fresh **Rose** Face **Mask** 3.4 oz, Fresh **rose** face **mask**, 3.5 oz, Fresh **Rose** Face **Mask**, **...**
www.nextag.com/fresh-**rose-mask**/search-html - 48k - Cached - Similar pages - Note this

Energizing **Rose Mask** - At-Home Facial Treatment **Masks** - iVillage **...**
With just a cup of plain yogurt and a few choice ingredients you can change the look and feel of your skin.
beauty.ivillage.com/skinbody/facecare/0,,96sswrnc,00.html - 54k - Cached - Similar pages - Note this

Indian **Rose Mask** - Craftbits.com Craft Project
Indian **Rose Mask** craft project, Free Craft Projects, Ideas, Crafts, Kids, Kids lesson plans, Wedding Crafts, Candle and Soap Making, Scrapbook, Crochet, **...**
www.craftbits.com/viewProject.do?projectID=764 - 71k - Cached - Similar pages - Note this

Sephora: Korres Wild **Rose Mask**: **Masks**
What it is: An instant brightening and illuminating vitamin C **mask**. What it is formulated to do: Suitable for all skin types, an instant brightening effect **...**
www.sephora.com/browse/product.jhtml?id=P155112&categoryId=C11245 - 63k - Cached - Similar pages - Note this

Sponsored Links

Costume **Rose**
Costume **rose**
Shop Target.com
www.Target.com

Rose Mask
Bargain Prices. Smart Deals.
Save on **Rose Mask**.
BizRate.com

Rose Mask For Less
Looking for **Rose Mask**?
Buy direct from sellers and save.
www.eBay.com

Mask Rose
Huge selection of
Mask Rose items.
Yahoo.com

Traffic Arbitrage

- Some search engines have their own sponsored keyword advertising markets.
- However, they also take part at other search engines' auctions and bring traffic back to their own markets.
  - Increasing the traffic that goes into their websites is an obvious way to increase their income.
- Examples: shopping.yahoo.com, nextag.com, bitraze.com, may themselves take part at sponsored keyword auctions of search engines such as Google and Yahoo.
  - The concept is motivated by the behavior of some participating websites of the AdSense market model of Google.

Traffic Arbitrage Strategy

- Auctioneer A_i bids for some slot on auction G_j to increase the traffic to his own search engine.
  - True value: $v^{Ai, Gj}$
  - Bidding price: $b^{Ai,Gj}$

Stability and Revenue

Theorem

Consider the traffic arbitrage model where all advertisers and arbitrageurs are following forward looking response function. We have:

1. There always exists a forward looking Nash equilibrium.
2. The model always converges to its forward looking Nash equilibrium.
3. In the forward looking Nash equilibrium, all the auctioneers' revenue will not be worse off in the presence of the traffic arbitrage behavior.

Click Arbitrage

- There are many affiliates undertaking advertising business. The commission depends on the traffic the affiliates bring to the clients' websites.
- The affiliate (maybe a search engine himself) can charge a fee for a click to his clients, at the same time to participate at the sponsored search auctions, paying less, to bring in potential consumers to the clients.
- If a potential user clicks on the advertisement on the search engines, he/she will be directed to the destination URL then redirected to the client's webpage

Click Arbitrage

- The search engine A furtively bids a slot from another search engine B's auction for A’s some participant i.
- Then A allocate the clicks won from B to the slot that participant i wins in his own auction to increase this slot's number of clicks.
  - If A pays less to B than it collects from i, the act would gain it extra revenue.

Interesting Observation

- Proposition:
  - If the auctioneer would apportion extra clicks among these K slots to maximally increase his revenue, he will apportion all the extra clicks to the first slot.

Click Arbitrage Strategy

Step 1:

Auctioneer A_i furtively represents the highest bidder in his own auction and participates in auction G_j.

Step 2:

Allocate all the clicks won from G_j to the first slot of his own auction.

Stability and Revenue

Theorem

Consider the click arbitrage model where all advertisers and arbitrageurs are following forward looking response function. We have:

1. There always exists a forward looking Nash equilibrium.
2. The model always converges to its forward looking Nash equilibrium.
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General Equilibrium Rationality

- Demand vs Supply
  - Market clearance: the equilibrium price will have all goods sold in the market.
  - The mechanism results in efficient allocation of resources.

- Fisher Model
  - Participants are buyers and sellers
    - Buyers have cash
    - Sellers have goods
  - Equilibrium:
    - All goods are sold
    - All Cashes are spent

Computational Paradigms for Equilibrium

Smith's Invisible Hand relies on the dynamic adjustment

Walras: Virtual Auction generates a sequence of tentative prices until the market clears.

Lange: Economic planner's computer can simulate whatever the market can do.

Scarf: Fixed point approach to solve the general equilibrium prices.

CGE: Equilibrium based parameter analysis to study policy issues.

Papadimitriou: Complexity Concept of PPAD

Bounded Rationality

- No infinite amount of resource is spent to achieve an optimal outcome by a bounded rational agent.

– Herbert Simon

Approximate Individual Rational Competitive Equilibrium

- Deng,Papaditriou,Safra (2002)
  - Approximate Individual Rationality: each agent maximize to approximate optimum when price is fixed.
  - Market clearance: for each goods, market almost clears.
  - Polynomial time algorithms are found for important special cases
    - In particular, integer cases when # of goods is a constant

Sponsored Search Market Input

- Advertisers
  - Bidding Price
  - Budget
- Search Engine
  - Ctr
    - Position related
    - Advert related
  - Quality Scores
  - …

Weakness in Theory

- Private Value
  - Known only to advertisers
    - Roughly calculated through evaluating #of clicks generated from online adverts and revenue received
- Evaluating GSP:
  - Where private values could be discovered through forward looking equilibrium
  - However, it is not accurate as practical outcomes deviate from theory by a large margin.

Practice vs Theory in Market Design

- eBAY:
  - Design: second price auction with a deadline similar to Vickrey
  - Reality: shown not to be equivalent to Vickrey
- GOOGLE
  - Design: GSP
  - Reality: not truthful
- Radio Bandwidth Auction
  - Design: much studied as a combinatorial auction
  - Reality: simultaneous split market auctions
    - Social utility not maximized but revenue is

What kind of data is available?

- Markets are highly electronically based.
  - All users' behavior are known in principle.

- What kind of data in the theory are available ?

Market Analysis based on Data

- Bids
- Budgets
- Market Models
  - CTRs
    - Different ways to obtain data and provide for analysis purpose.

Walrasian Equilibrium

- Items of non-zero values are all sold
- Items leftover have price zero

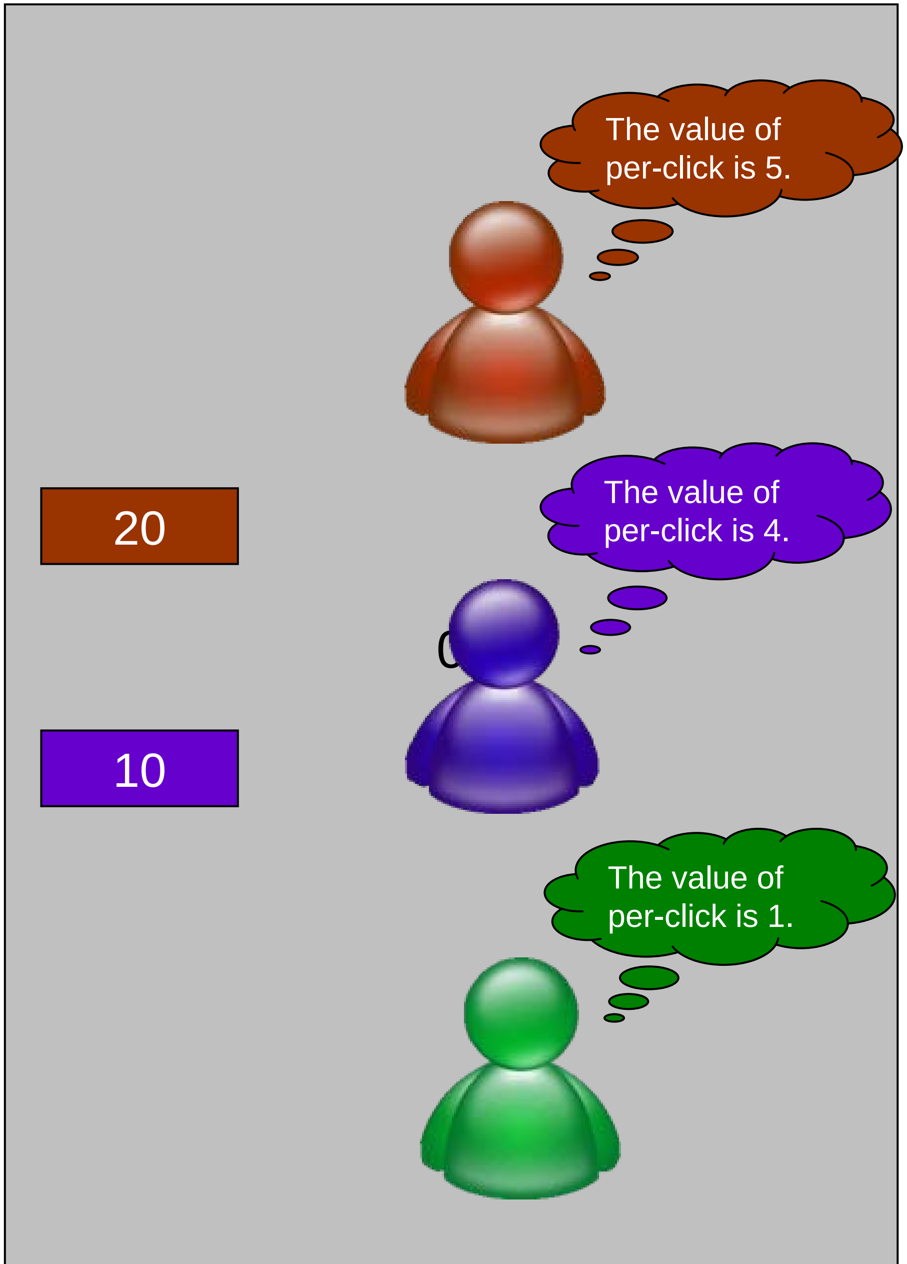
- All participants maximize their revenue under th price vector

Demange and Gale

- Concept of minimum market equilibrium price

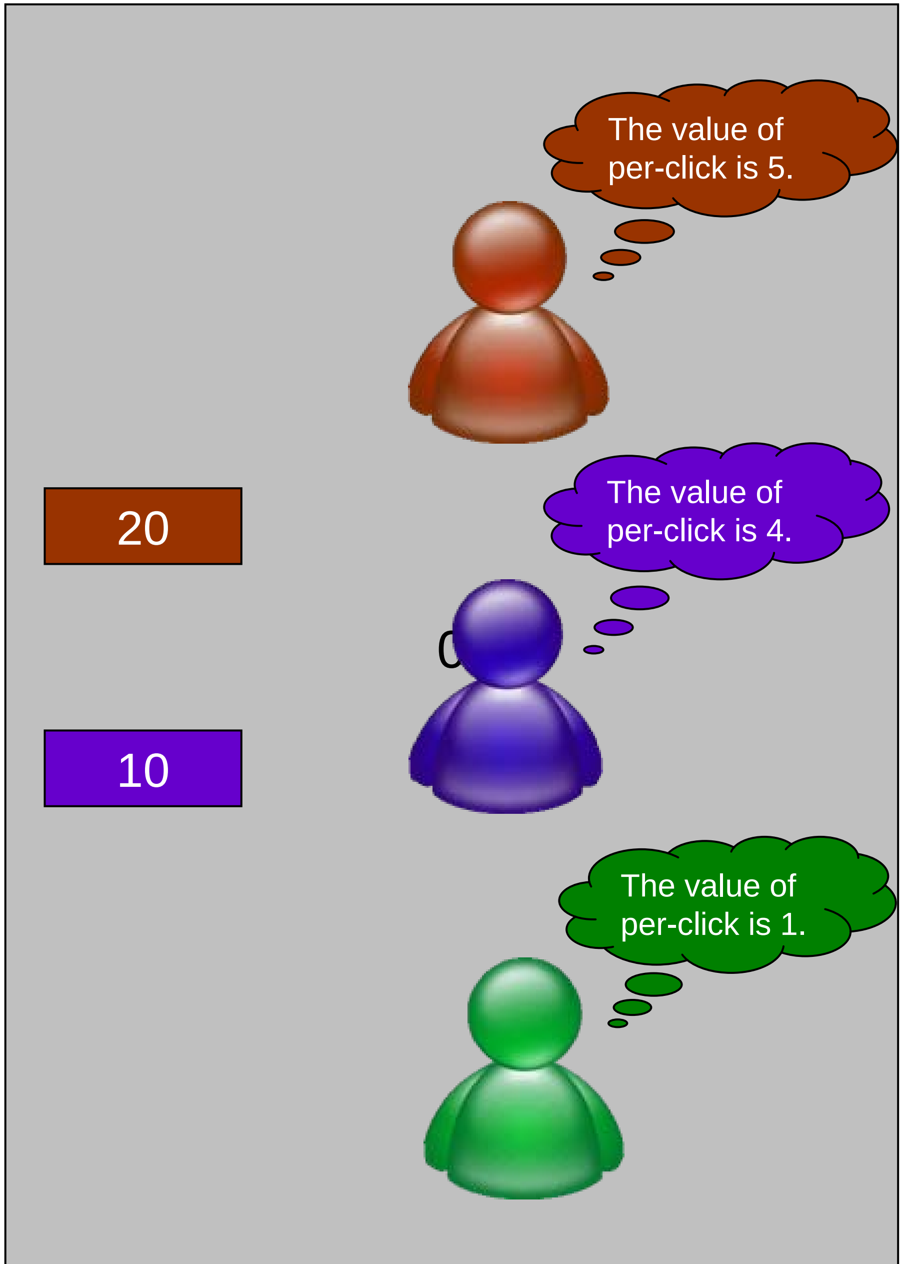
- the work of Edelman et al, and Varian of envy free as well as symmetric nash equilibrium are all equivalent to the minimum equilibrium.

Market Price of both slots at 0



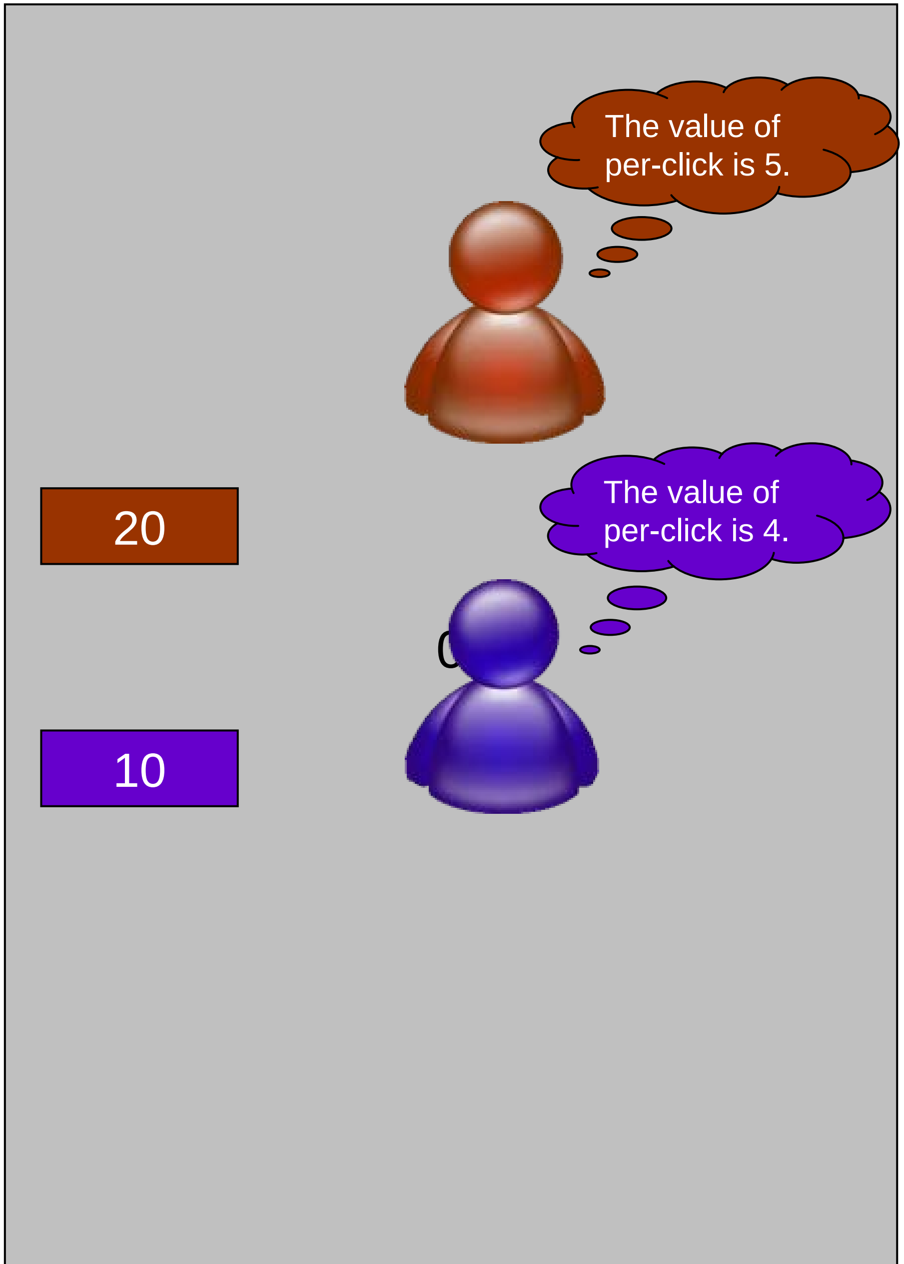
- Blue prefers the top slot
- So do all other two
- Market does not clear.

Prices at least one



- When any price is less than one, three persons would want two slots which is impossible to give two slots to three persons
- Prices larger than one
  - Green drops out

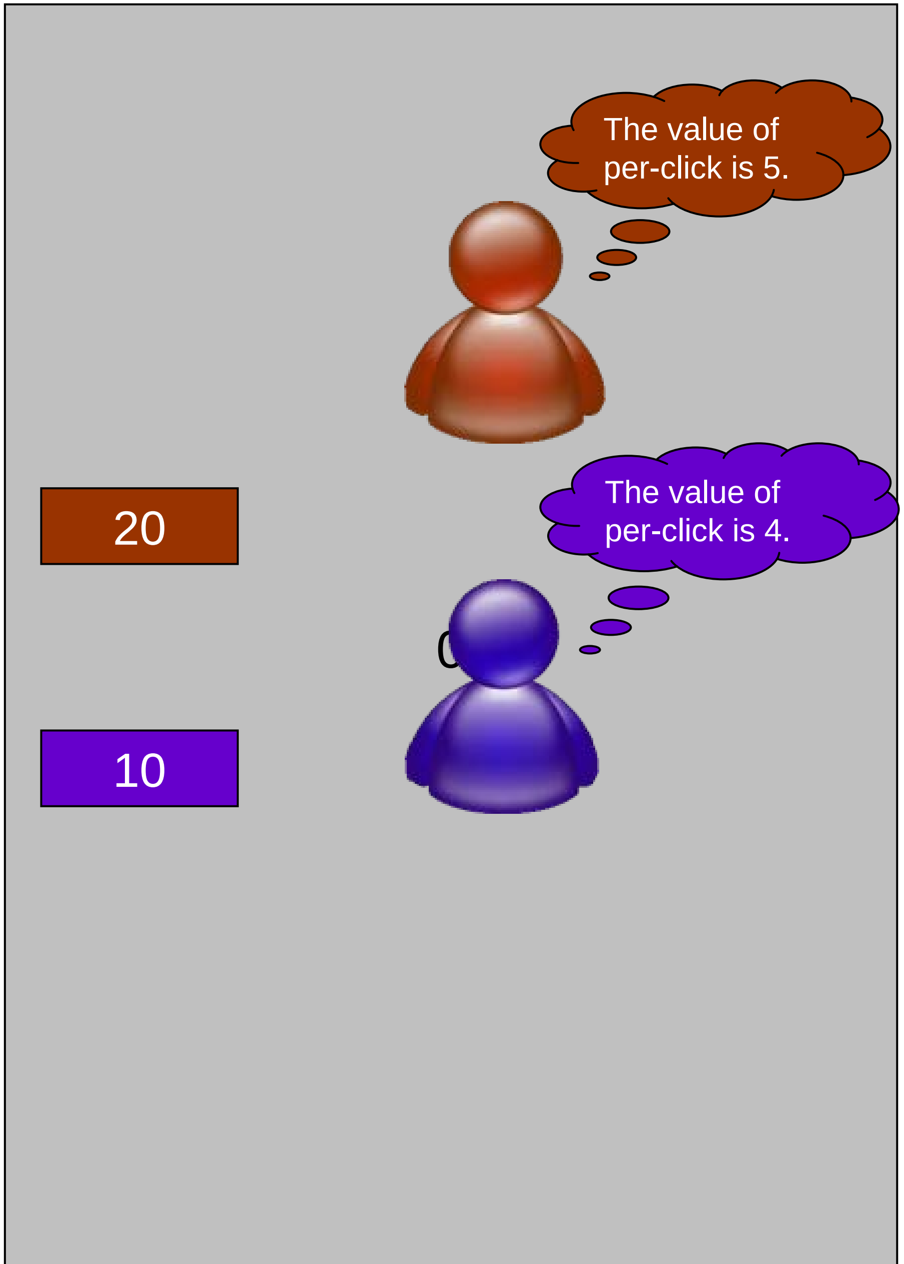
Prices at 1+



- When any price is less than one, three persons would want two slots which is impossible to give two slots to three persons

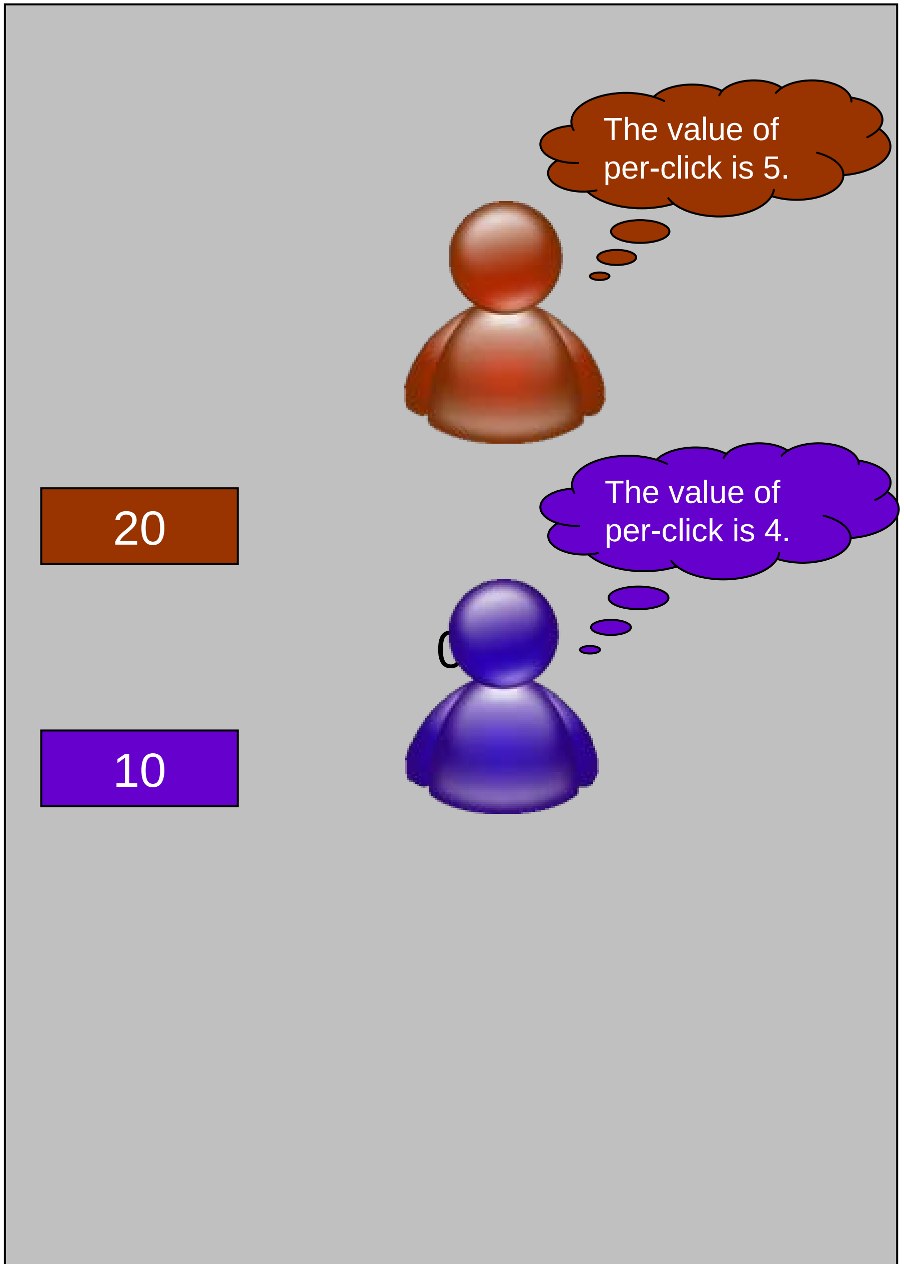
- Prices larger than one
  - Green drops out

Both Prices at 1+



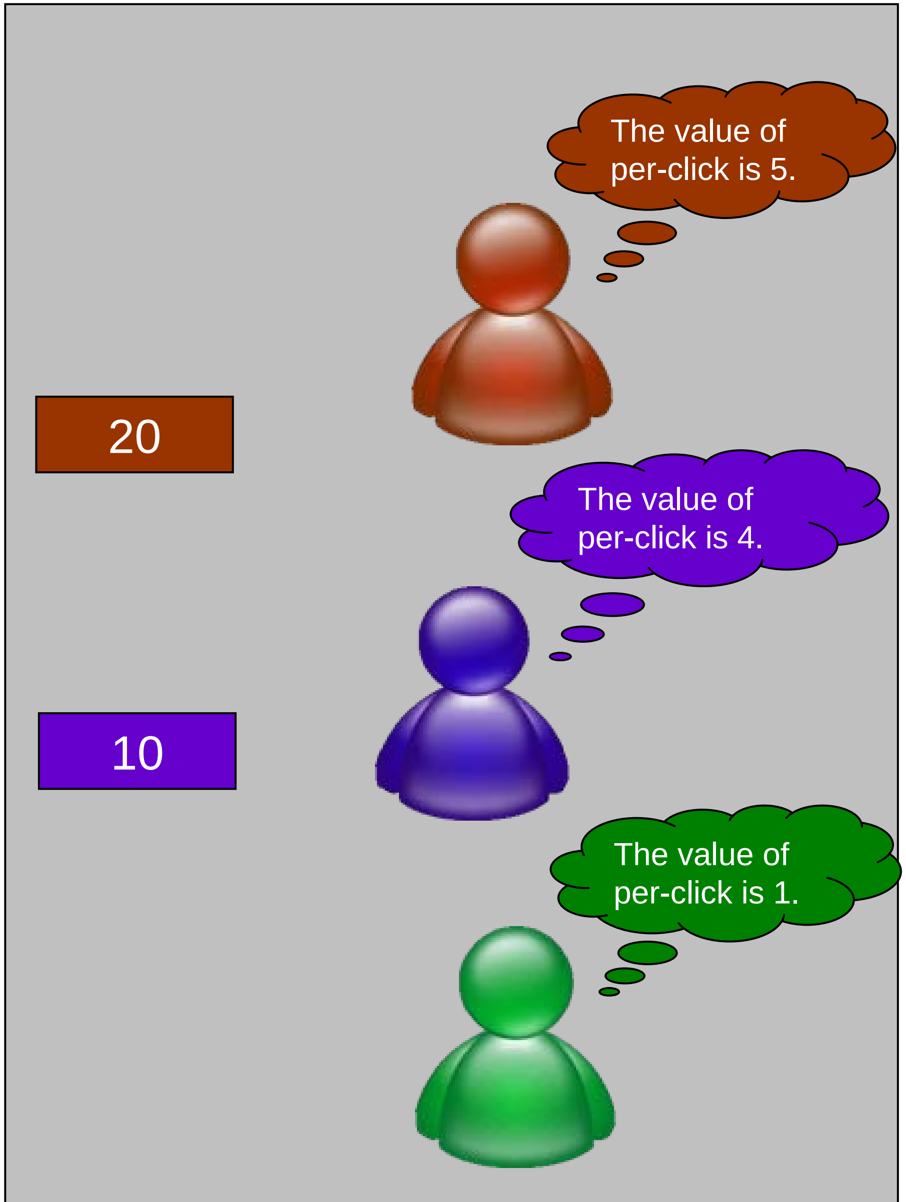
- Both Brown and Blue prefer the top slot than the bottom slot.
- The seller could price top slot at a higher price.

Prices at (2.5, 1)



- Brown’s utilities:
  - (5-2.5)*20=50
  - (5-1)*10=40
  - Prefers top slot
- Blue’s utilities
  - (4-2.5)*20=30
  - (4-1)*10=30
  - Indifferent at two slots
- Solution: a matching.
  - Red gets top
  - Blue gets bottom
- Market clears.

Priced at (2.5,1)



- Brown's utilities:
  - (5-2.5)*20=50
  - (5-1)*10=40
- Blue's utilities
  - (4-2.5)*20=30
  - (4-1)*10=30
- Green's utility: 0
- Solution: a matching.
  - Red gets top
  - Blue gets bottom
  - Green gets noting
- Everyone is happy,
- Market clears.

Necessity to Change

- GSP's weakness:
  - Cannot handle budget conditions
  - Quite restrictive and cannot extend well into general settings

What theory to use in practice?

- Market equilibrium formulation
  - Competitive equilibrium
    - Market clearance
      - Prices change till all slots are assigned.
  - Recently: A polynomial time algorithm (Chen Ning and D).
  - A closely related stable solution concept:
    - Aggarwal, Gagan and Muthukrishnan, S. and P疝, D疱id and P疝, MartinGeneral Auction Mechanism for Search Advertising. (2009 www)

Other important issues

- Multiple word biddings
  - Similar but larger set of problems each with a different bids, a total budget.
- User behavior?
- Bidder coordination?